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FINAL TECHNICAL REPORT

On

Surface Boundary Conditions and Related Technical Issues Concerning Burnett Equations for 2-D Hypersonic Flow

Prepared by

Xiaolin Zhong, Assistant Professor

Mechanical, Aerospace and Nuclear Engineering Department

University of California, Los Angeles, CA 90024

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Air Force Office of Scientific Research
Bolling Air Force Base, DC 20332-6448

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1 Summary

Research in the area of studying steady and transient rarefied hypersonic flow by using the Burnett equations and developing accurate and efficient numerical methods for both the Navier-Stokes equations and the Burnett equations were conducted in the two-year period (11/15/91 - 11/14/93) of this AFOSR grant.

The activities and accomplishments of the research in this period are summarized as follows:

- 1. We have developed a new method in treating boundary conditions for the Burnett equations, and computed Burnett solutions for two-dimensional hypersonic flow past a cylinder to evaluate the effects of the surface boundary conditions. The results show that the flow-field solutions of the Burnett equations can be uniquely obtained by using the new boundary conditions, and the Burnett solutions with the first-order slip conditions agree better with DSMC results than the Navier-Stokes solutions do for the case of Knudsen number 0.2. The main difference between the Navier-Stokes solutions and the Burnett solutions are in the temperature flow fields.
- 2. We have developed finite-volume computer codes for the high-order ENO schemes which can be uniformly up to fourth-order accurate for solving the Navier-Stokes equations (as well as the Burnett equations) for two-dimensional transient and steady hypersonic flow. The third-order finite-volume ENO code was used to study unsteady hypersonic shock-shock interference heating on a circular cylinder. The computational results show that the interference-heating flow is inherently unsteady and the inherent unsteadiness has a strong effect on such flow parameters as surface heating rates.
- 3. We have derived the three-dimensional components of the Burnett stress and heat-flux terms from the general tensor forms and extended our numerical methods for solving plane two-dimensional augmented Burnett equations to axisymmetric augmented Burnett equations. We have obtained, for the first time, the numerical solutions of the augmented Burnett equations for axisymmetric hypersonic flow past spherical blunt bodies. The comparison of the results shows that the Burnett solutions predict a much thicker shock wave than the Navier-Stokes equations as expected for transitional flow.

These research results will contribute the understanding of hypersonic flow phenomena in the transitional flow regime, though more work is needed to validate the Burnett equations for wall-bounded multidimensional hypersonic flow.

2 Introduction

The goal of this research is to study steady and transient three-dimensional hypersonic flow phenomena which are important to the development of future hypersonic vehicles. To reach this goal, it is necessary to develop nonequilibrium flow models for the thermodynamics effects of internal molecular excitation and to develop accurate and computationally efficient methods for computing complex two- and three-dimensional steady and transient hypersonic flow.

Currently, hypersonic rarefied gas flow in the free-molecule regime can be studied by using the direct simulation Monte Carlo (DSMC) technique. Though it is possible to use the DSMC technique to compute some simple flow fields in the continuum and transitional regimes where the Navier-Stokes solutions are valid, three-dimensional transient hypersonic flow in the transitional regime is computationally too expensive for the DSMC approach. Therefore, the continuum approach based on the Navier-Stokes equations have been used for studying the flow, but the Navier-Stokes equations breakdown as the Knudsen numbers increase. It is not very clearly understood how far in terms of Knudsen number the Navier-Stokes equations can be used to studying hypersonic flow phenomena and how the flow structure and parameters are different when the Navier-Stokes equations begin to breakdown.

The research project of this two-year AFOSR grant was focused on using the Burnett equations to extend the continuum approach for studying steady and transient rarefied hypersonic flow in the continuum transitional regime, and to develop accurate and efficient CFD predictive tools for these studies. The advantage of using the Burnett equations is its computational efficiency over the DSMC technique and its accuracy over the Navier-Stokes equations in the transitional flow regimes. The main applications of the present research are the Burnett equations approach can be used to

- 1. study when and how the continuum approach breaks down for complex steady and transient hypersonic flow field,
- 2. study transient hypersonic flows in transitional flow regime. Examples of the flow include hypersonic wall-bounded turbulence structure and stability, hypersonic shock

structure, and shock wave/boundary layer interaction under nonequilibrium transitional flow conditions.

The objectives of the research in the period of this grant were to study the issue of boundary conditions for the Burnett equations, to study wall-bounded multidimensional flows, and to develop accurate and efficient numerical methods for steady and transient two-dimensional hypersonic flow computations. Specifically, The following research problems have been investigated:

- 1. Developed new method in treating boundary conditions for the Burnett equations, and computed Burnett solutions for two-dimensional hypersonic flow past a cylinder to evaluate the effects of surface boundary conditions.
- 2. Evaluated and applied the high-order accuracy essentially nonoscillatory (ENO) schemes to study steady and transient hypersonic flow efficiently and accurately. The unsteady shock interference problem was studied by using the third-order ENO schemes.
- 3. Developed numerical methods and computer codes for solving the Burnett equations for axisymmetric flows, and studied axisymmetric hypersonic flow past a sphere by using the Burnett solutions.

In the following sections, the highlights of these research activities are presented; the details can be found in the references cited there.

3 Research Highlights

3.1 Boundary Conditions for the Burnett Equations and Applications to Hypersonic Flow Computations

Much progress has been made in using the Burnett equations for rarefied hypersonic flow. Investigations in mainly one-dimensional Burnett solutions for shock-wave structure have shown that it is possible to advance the continuum approach into the more rarefied transition flow regime by using the Burnett equations. Recently, Zhong, MacCormack and Chapman have obtained the first known 2-D Burnett solutions for hypersonic flow past blunt leading edges. However, it is still uncertain how to formulate boundary conditions for the higher-order Burnett equations. Furthermore, few comparative studies have been performed to validate the Burnett equations in multidimensional applications.

Therefore, we investigated on the issues of boundary conditions, and the details of the results can be found in Ref. [2]. The major results are:

- 1. We proposed a new method of specifying additional boundary conditions for the Burnett equations. The new method requires the same number of physical slip conditions as those required by the Navier-Stokes equations. The additional boundary conditions for the higher-order terms in the Burnett equations are derived from the corresponding Navier-Stokes solutions. The Burnett solutions obtained by using the new boundary condition formulation are accurate up to the Burnett level of approximation to the Boltzmann equation.
- 2. We obtained numerical solutions of the two-dimensional augmented Burnett equations with the new formulation of boundary conditions for Mach 10.95 flow past a cylinder. The range of free-stream Knudsen numbers for the cases is from 0.02 to 0.6. For each case, we have obtained the solutions of the Navier-Stokes equations, the augmented Burnett equations with the first-order Maxwell/Smoluchowski slip conditions, and the augmented Burnett equations with the second-order Schamberg slip conditions. A detailed comparative study on the Burnett, Navier-Stokes, and DSMC solutions are

carried out for one of the cases of free-stream Knudsen number 0.2. Meanwhile, for other cases, the Burnett solutions are compared with the Navier-Stokes solutions because DSMC solutions are not available for those cases. These results can be used for future comparisons with DSMC results when the latter are available.

As an example of the results, the flow-field temperature contours of the DSMC and Navier-Stokes results are compared in Figure 1, and the temperature contours of the DSMC and Burnett results are compared in Figure 2. All the contours show that the Burnett solutions agree better with the DSMC results than the Navier-Stokes solutions. The results show that the flow-field solutions of the Burnett equations can be uniquely obtained by using the new boundary conditions, and the Burnett solutions with the first-order slip conditions agree better with DSMC results than the Navier-Stokes solutions do, but the Burnett solutions with the Schamberg second-order slip conditions seem to be inaccurate for Knudsen numbers above 0.2.

3.2 Transient Hypersonic Flow Computations Using the ENO Schemes

The complexity of steady and transient three-dimensional hypersonic flow fields require efficient and accurate numerical methods. An example of these flow problems is the shock-shock interference heating problem, which is a critical problem in the development of future hypersonic vehicles because the most intense local heating rates on the vehicle are expected to be on cowl lips caused by shock-shock heating^[3]. The ability to accurately and efficiently predict these flow characteristics and transient flow phenomena is critical for successful design.

For computing this kind of hypersonic flow fields, high-order accurate shock capturing numerical schemes are required. However, current high-resolution shock capturing schemes, such as the TVD (total variation diminishing) schemes, are not uniformly higher-order accurate. These schemes necessarily reduce to first-order accuracy at the local extrema of solutions while maintain second-order accuracy in smooth regions. Therefore, they may not be uniformly accurate enough for computing complex transient flow. In recent years, a class of Essentially Non-Oscillatory (ENO) schemes, which are able to achieve uniformly high-

order accuracy with shock capturing capability, has been introduced by Harten and Osher of UCLA^[4]. Because of their uniformly high order accuracy, the ENO schemes may be particularly appropriate for complicated viscous flow fields with hypersonic shock interaction. The ENO schemes had been successfully applied to many 2-D inviscid supersonic flows, but not to the viscous compressible flow with non-trivial geometries and solid walls.

The potential advantage of these high order ENO schemes give us the impetus to use the schemes to study viscous shock interaction problems for high-altitude hypersonic flow. The details of the results can be found in Refs. [5, 6]; the highlights of this aspect of the research are

- 1. We have developed finite-volume computer codes for the new ENO schemes which can be uniformly up to fourth-order accurate for solving the Navier-Stokes equations (as well as the Burnett equations) for two-dimensional transient and steady hypersonic flow. The new schemes have been validated for many test problems, including supersonic boundary-layer, supersonic shock-wave/boundary-layer interaction, and simple model wave and heat equation.
- 2. The third-order finite-volume ENO code was used to study unsteady hypersonic shockshock interference heating on a circular cylinder. Time-accurate high-resolution numerical solutions of the Navier-Stokes equations for a type IV hypersonic shock-wave interference heating problem have been obtained by using the third-order accurate ENO scheme. The computational results show that the interference-heating flow is inherently unsteady and the inherent unsteadiness has a strong effect on such flow parameters as surface heating rates.

Figure 3 shows the comparison of instantaneous surface heating rates and surface pressure with experimental results by Wieting and Holden^[7,8]. The pressure and heat transfer rates along the surface are normalized by the corresponding values of the same free stream without interference heating. Figure 3 shows that the numerical results for surface heating rates compare reasonably well with the experimental results. The unsteady effect on the surface heating rates is more significant with the maximum heating varying between 8 and 20 due to the oscillation, which shows that it is important to study the unsteady flow field with time accuracy in order to predict the interference

heating rates correctly.

The unsteady features of the flow field and the mechanism of the inherent unsteadiness of the type IV interference heating flow are more clearly shown by examining the same flow field temperature contours at consecutive moments of time. The "zoomed" temperature contours at consecutive moments of time with 1000 time steps between each set of contours are shown in Figures 4 to 6. The unsteady mechanism can be observed in the numerical solutions of these temperature contours. The flow is inherently unstable even when the free-stream flow conditions are constant.

3. In order to validate the accuracy of the ENO schemes viscous flow simulations, we have also done grid-refinement studies on the ENO schemes to validate the ENO schemes for steady and unsteady inviscid and viscous flows simulations. We confirmed the finding of Rogerson and Meiburg^[9] that the accuracy of original ENO schemes reduced as the higher order ENO reconstruction are used. Modifications suggested by Shu^[10] have been used and validated for high-order accuracy of the schemes. The ENO schemes are found to work very well for high-order transient viscous flow simulations.

We are currently using the ENO schemes to study transient wall-bounded hypersonic flows; the results will be presented in the upcoming AIAA meetings.

3.3 Burnett Solutions for Axisymmetric Hypersonic Flow Past a Sphere

As a first step in extending our two-dimensional Burnett results to three-dimensional flow studies. We have extended the Burnett equations to axisymmetric hypersonic flow.

The details of the results will be presented in the upcoming AIAA/ASME 6th Joint Thermophysics and Heat Transfer Conference^[2]. The major results are:

1. We have derived the three-dimensional components of the Burnett stress and heat-flux terms from the general tensor forms; then we extended our numerical methods for solving planar two-dimensional augmented Burnett equations to axisymmetric augmented Burnett equations.

2. We have obtained, for the first time, the numerical solutions of the augmented Burnett equations for axisymmetric hypersonic flow past a sphere. The augmented Burnett and Navier-Stokes solutions show similar trend as the corresponding planar two-dimensional cases. The flow-field temperature contours of the augmented Burnett and Navier-Stokes results are compared in Figure 7. The contours show that the Burnett solutions predict a much thicker shock than the Navier-Stokes equations. The temperature contours show significant difference between the augmented Burnett results and the Navier-Stokes results. Compared with the plane two-dimensional results in Ref. [2], the axisymmetric Navier-Stokes and Burnett solutions predicts a much smaller stand-off shock distance.

4 Conclusions

- 1. A new method of formulating the additional boundary conditions for the Burnett equations is presented. The new boundary conditions for the Burnett equations worked satisfactorily in the test cases.
- 2. The Burnett equations with first order slip conditions seem to be able to improve the Navier-Stokes equations in two dimensions in the continuum transition regime, though more validation work is needed and is currently underway.
- 3. The high-order ENO schemes have been used to obtain time-accurate solutions of the Navier-Stokes equations to study the unsteady hypersonic shock-shock interference heating on a cylinder. The results show that the inherent unsteadiness has a strong effect on surface heating rates. The ENO schemes are found to work well for numerical simulations of transient hypersonic flow.
- 4. The three-dimensional components of the Burnett stress and heat flux terms have been derived and numerical methods for solving the axisymmetric Burnett equations have been developed. The Burnett solutions for axisymmetric flow have been obtained for the first time by using our newly developed axisymmetric Burnett computer code.

5 Acknowledgements

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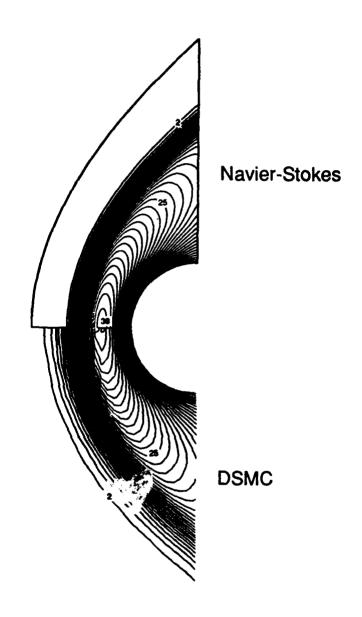


Figure 1: Flow-field temperature (T/T_{∞}) contour comparison between the Navier-Stokes solutions and the DSMC results $(M_{\infty} = 10.95, \text{ and } Kn_{\infty} = 0.2)$.

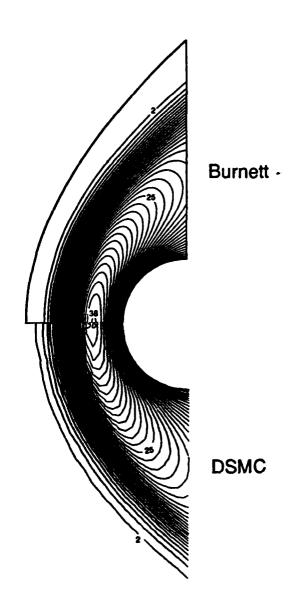


Figure 2: Flow-field temperature (T/T_{∞}) contour comparison between the augmented Burnett solutions and the DSMC results $(M_{\infty}=10.95, \text{ and } Kn_{\infty}=0.2)$.

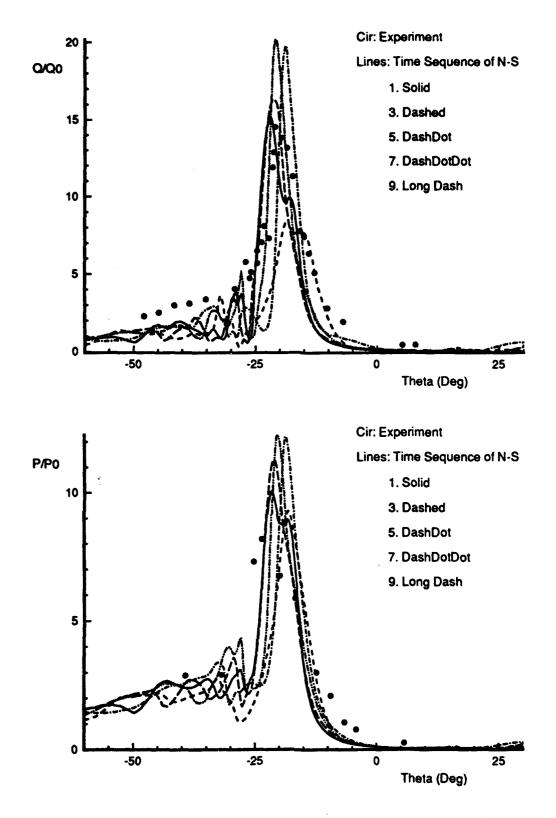


Figure 3: Instantaneous surface pressure and heat transfer rates for type IV shock interference flow at $M_{\infty}=8.03$.

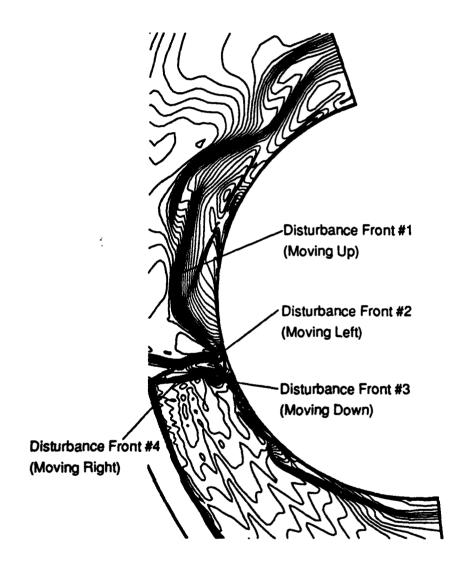


Figure 4: The 1st of the nine consecutive temperature contours for the unsteady type IV shock interference, $M_{\infty}=8.03$, Time = 1.527×10^{-4} sec.

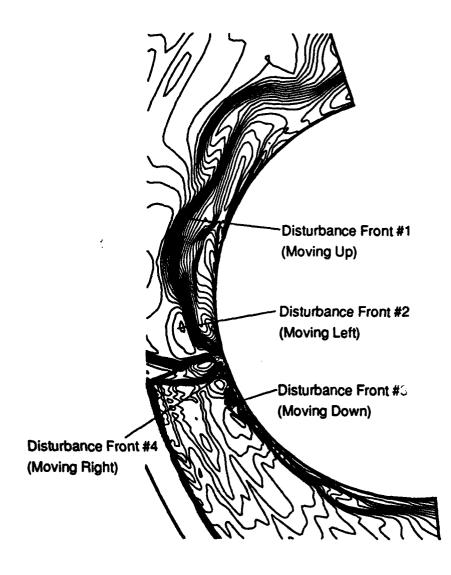


Figure 5: The 5th of the nine consecutive temperature contours for the unsteady type IV shock interference, $M_{\infty}=8.03$, Time = 1.679×10^{-4} sec.

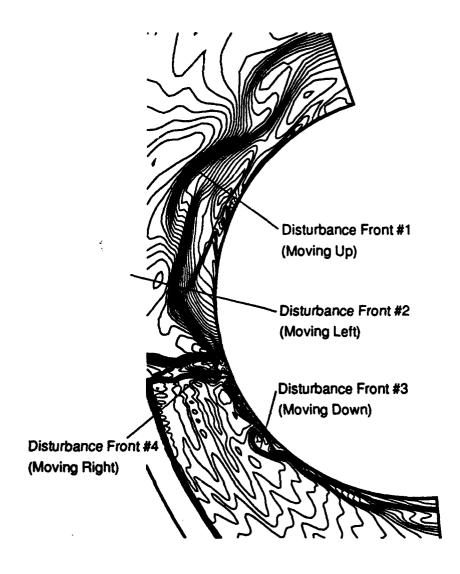


Figure 6: The 9th of the nine consecutive temperature contours for the unsteady type IV shock interference, $M_{\infty}=8.03$, Time = 1.822×10^{-4} sec.

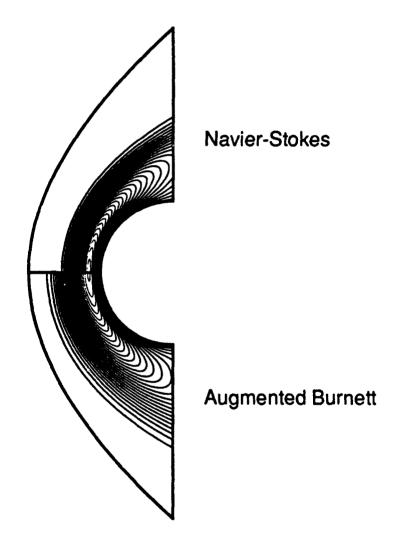


Figure 7: Flow-field temperature contours comparison (Case 2: $M_{\infty}=10.95$, $Kn_{\infty}=0.2$).